

Turing Machine II

Zeyu Mi

2023-12-20



Adapted From:
IALC 8.2,8.3.1,9.1,9.2

~~What problems can a computer solve?~~



~~What languages can a computer recognize?~~



What languages can TM recognize?

How we investigate computability?

2.1 Discussion on Problems

Part1. Intro & Set theory(I) : Basics & Formal Language.

Part2. Set Theory(II): Axiom system & Cardinality.

Part3. Capture Structures: Binary Relation & Function

2.2 Discussion on Computation

Part1. Turing Machine Basics.

Part2. Variants of Turing Machine. Church-Turing Thesis.

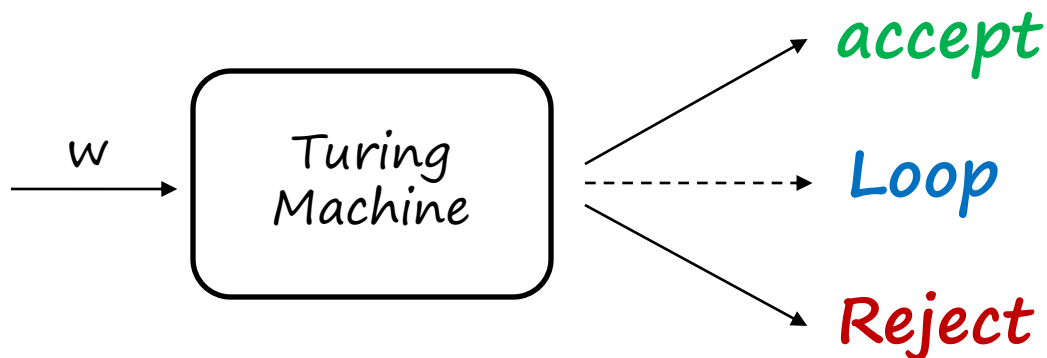
2.3 Discussion on computability

Part1. The Language of Turing Machine. R & RE

Part2. Undecidability.

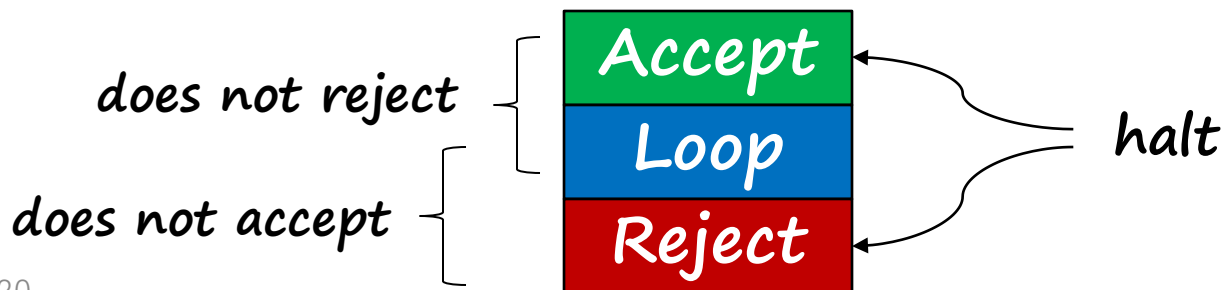
Output of TM

- For a certain input string, the output of a TM can be one in three kinds:
 - The TM **accepts** this string.
 - The TM **rejects** this string.
 - Rather than accepting or rejecting, the TM **loops** forever on the input string and never stops.



Very Important Terminology

- Let M be a Turing machine and let s be a string.
 - M **accepts** s if it enters an accepting state when running on s
 - M **rejects** s if it enters a rejecting state when running on s .
 - M **loops infinitely** on s when running on s if it enters neither an accepting nor a rejecting state.
 - M **does not accept** s if it either rejects s or loops on s .
 - M **does not reject** s if it either accepts s or loops on s .
 - M **halts** on s if it accepts s or rejects s .



More Details of RE

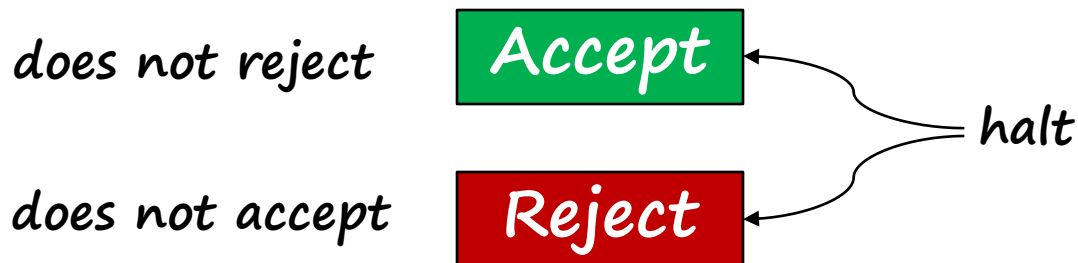
- We've known that for a certain language L , if there exists a TM M such that $L = L(M)$, then we call L is a **recursively enumerable language(递归可枚举语言)**, or **RE**.
- So for any RE L , a TM M that $L(M) = L$, and any string w
 - If $w \in L$, M accepts w in finite steps.
 - If $w \notin L$, M **does not accept** w , it means M rejects w or **loops forever**
- We also call the TM M a **recognizer** for the language L , and L is **Turing-recognizable(图灵可识别的)**.

More Details of RE

- In other words, as a recognizer, M will tell you correct on every correct input, but M is not necessary to tell you wrong on every wrong input.
 - You can not determine the input is correct or wrong until M halts.
- But a problem is solvable(computable) if the computation process finishes in finite steps.

Decider

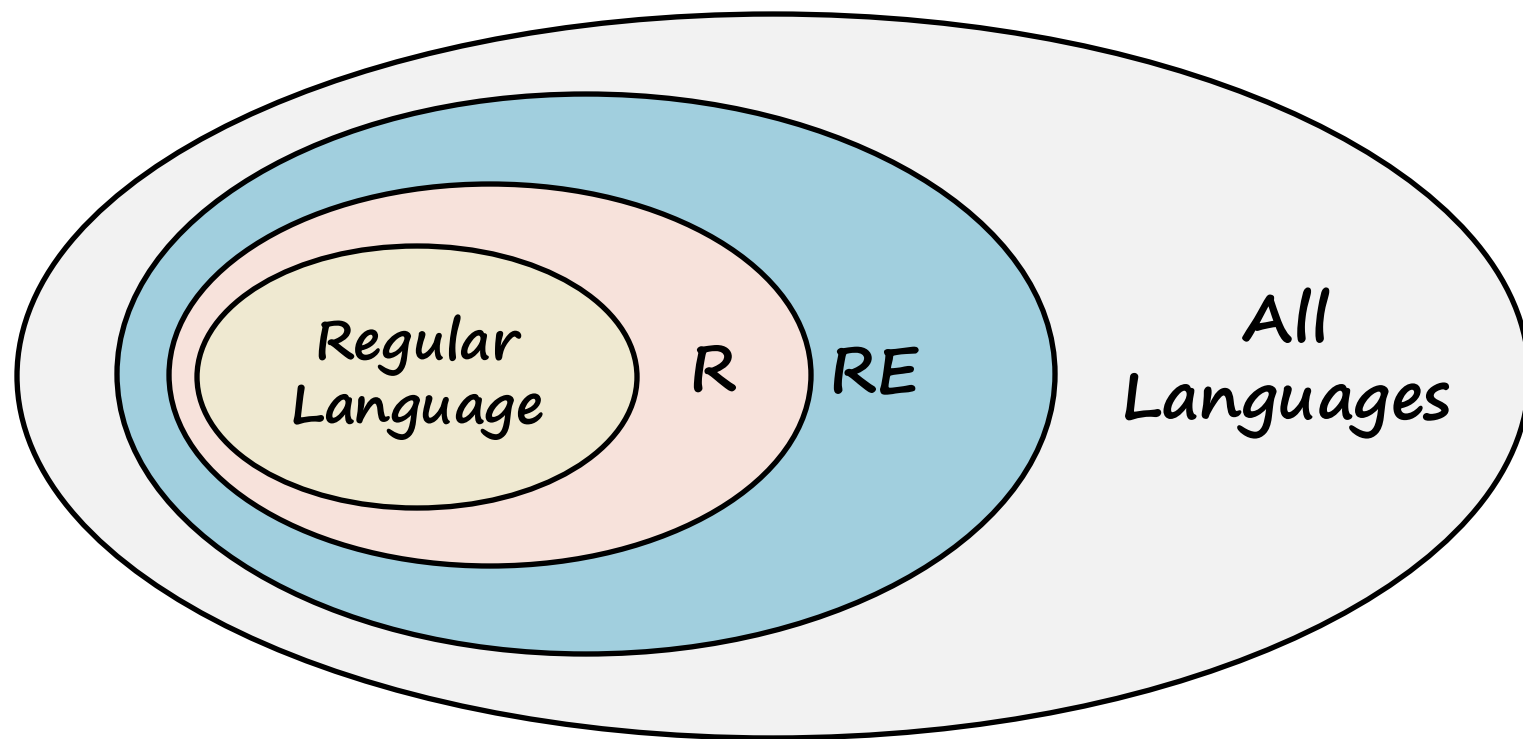
- Some Turing machines always halt; they never go into an infinite loop.
- If M is a TM and M halts on every possible input, then we say that M is a decider.
- For deciders, accepting is the same as not rejecting and rejecting is the same as not accepting.



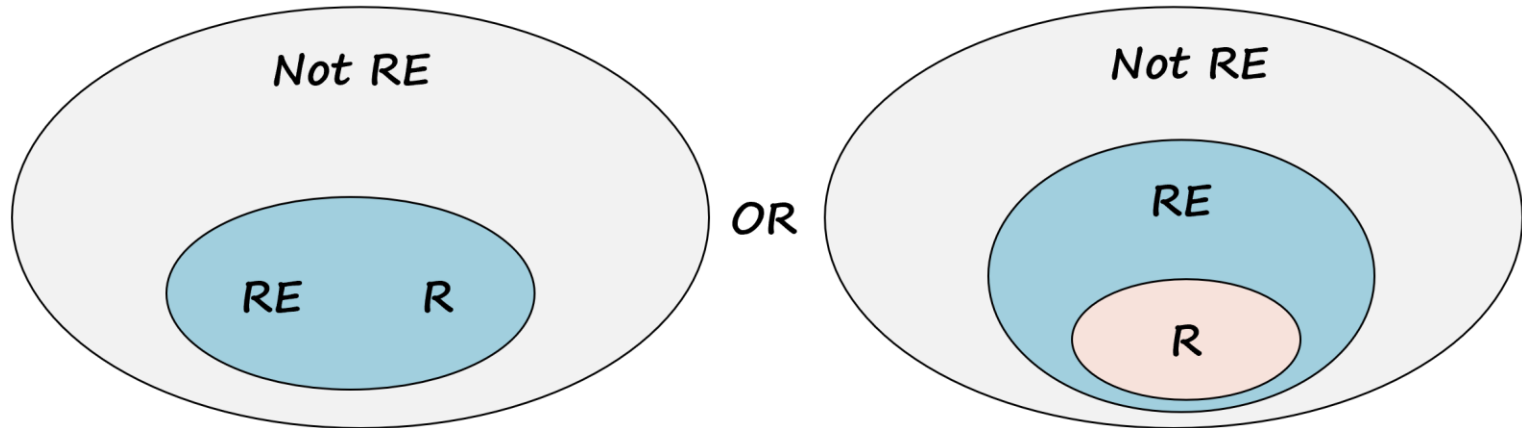
Recursive Language

- A language L is **recursive(递归的)** if there's a TM M such that:
 - If $w \in L$, M accepts w in finite steps.
 - If $w \notin L$, M rejects w in finite steps.
- We also call the TM M a **decider** for the language L , and L is **Turing-decidable(图灵可判定的)**.
- Decidable problems, in some sense, are that can definitely be “solved” by a computer.
- Recursive Language is a subset of RE language

Language Hierarchy



Question:
 $RE=R?$



Universal Turing Machine

通用图灵机

An Observation

- When we've been discussing Turing machines, we've talked about designing specific TMs to solve specific problems.
 - TM for $0^n 1^n$
- Does this match your real-world experiences? Do you have one computing device for each task you need to perform?
- Or can we make a “**reprogrammable** Turing machine?”

A TM Simulator

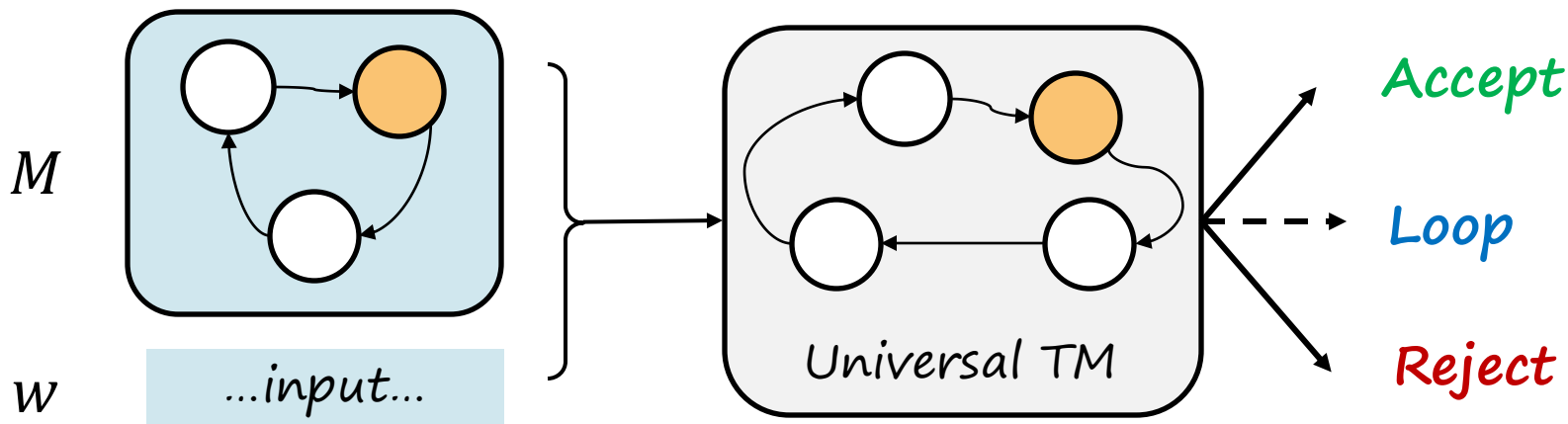
- It is possible to program a TM simulator on an unbounded-memory computer.
- We could imagine it as a method

boolean simulateTM(TM M , string w)

- with the following behavior:
 - If M accepts w , then *simulateTM*(M , w) returns true.
 - If M rejects w , then *simulateTM*(M , w) returns false.
 - If M loops on w , then *simulateTM*(M , w) loops infinitely.

A TM Simulator

- Anything that can be done with an unbounded-memory computer can be done with a TM
- This means that there must be some TM that has the behavior of this *simulateTM* method.



The Universal Turing Machine

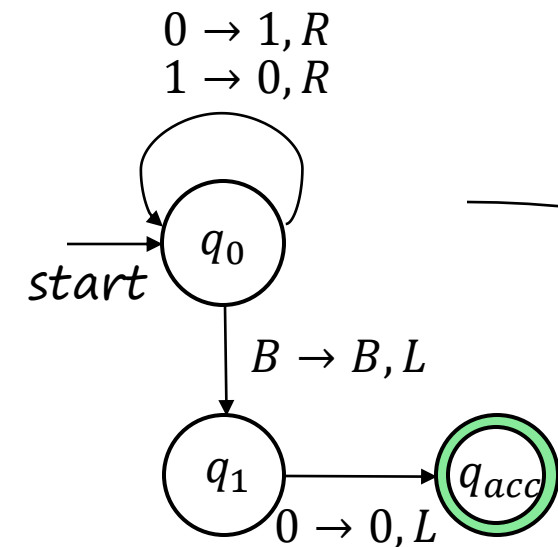
- **Theorem (Turing, 1936):** There is a Turing machine U_{TM} called **the Universal Turing Machine** that, when run on an input of the form $\langle M, w \rangle$, where M is a Turing machine and w is a string, simulates M running on w and **does whatever** M does on w (accepts, rejects, or loops).
- U_{TM} behaves as follows:
 - If M accepts w , then U_{TM} accepts $\langle M, w \rangle$
 - If M rejects w , then U_{TM} rejects $\langle M, w \rangle$
 - If M loops on w , then U_{TM} loops on $\langle M, w \rangle$

Encoding Input with binary

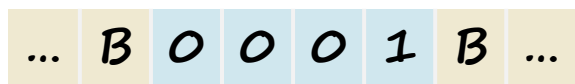
- Input string may contains any possible character in the input alphabet of M
- But we know everything on your computer is a string over $\{0, 1\}$
- We can let the input alphabet to be $\{0,1\}$
- It not necessary to limit the alphabet as $\{0,1\}$, but only for simplicity.

The Universal Turing Machine

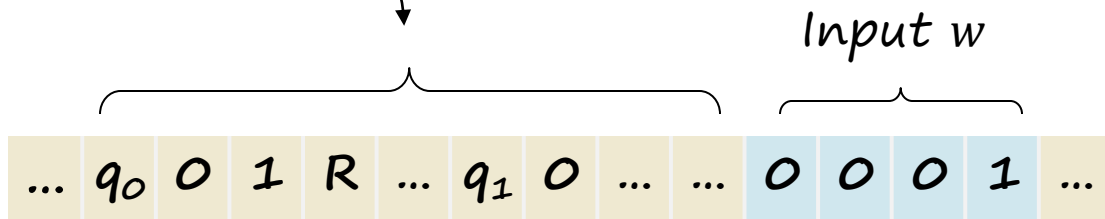
Machine M



TM as Input?



2023/12/20



Encoding TM

- In order to take a Turing Machine as an input, we need to encoding the TM. Similarly, we shall encode TM with binary.
- We first assign integers to the states, tape symbols and directions
 - We assume the states are $q_1, q_2, \dots, q_k$ for some k . q_1 is the **input state** and q_2 is the **only accept state**. (Is it right?)
 - The tape symbols are $X_1, X_2, \dots, X_m$ for some m . X_1, X_2, X_3 are **0, 1, B** respectively.
 - Refer to direction **L as D_1** , **R as D_2**

Encoding TM

- After assigning each state, symbol and direction an integer, we can encode the transition function δ .
- Suppose one transition rule is $\delta(q_i, X_j) = (q_k, X_l, D_m)$, we shall code this rule by the string $0^i 10^j 10^k 10^l 10^m$.
 - Notice i, j, k, l, m are at least one, so there're no occurrences of two or more consecutive 1's with in the code for a transition
- So let C_i donate the code for the i th transition rule, we can encode the whole TM as:
 - $C_1 11C_2 11C_3 11 \dots 11C_n$

Example of Code for TM

- Let a TM: $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, \{\delta\}, q_1, B, q_2)$
 - $X_1 = 0, X_2 = 1, X_3 = B, D_1 = L, D_2 = R$
- Transition Function δ :
 - $\delta(q_1, 1) = (q_3, 0, R)$
 - $\delta(q_3, 0) = (q_1, 1, R)$
 - $\delta(q_3, 1) = (q_2, 0, R)$
 - $\delta(q_3, B) = (q_3, 1, L)$

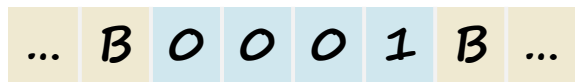
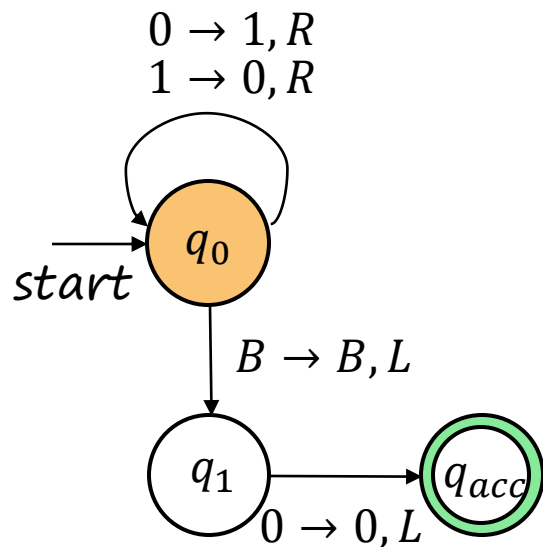
Example of Code for TM

- Let a TM: $M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}, \{\delta\}, q_1, B, q_2)$
 - $X_1 = 0, X_2 = 1, X_3 = B, D_1 = L, D_2 = R$
- Transition Function δ :
 - $\delta(q_1, 1) = (q_3, 0, R) \Rightarrow \delta(q_1, X_2) = (q_3, X_1, D_2) \Rightarrow 0100100010100$
 - $\delta(q_3, 0) = (q_1, 1, R) \Rightarrow \delta(q_3, X_1) = (q_1, X_2, D_2) \Rightarrow 0001010100100$
 - $\delta(q_3, 1) = (q_2, 0, R) \Rightarrow \delta(q_3, X_2) = (q_2, X_1, D_2) \Rightarrow 00010010010100$
 - $\delta(q_3, B) = (q_3, 1, L) \Rightarrow \delta(q_3, X_3) = (q_3, X_2, D_1) \Rightarrow 0001000100010010$
- Code for this TM:

01001000101001100010101001001100010010010100110001000100010010

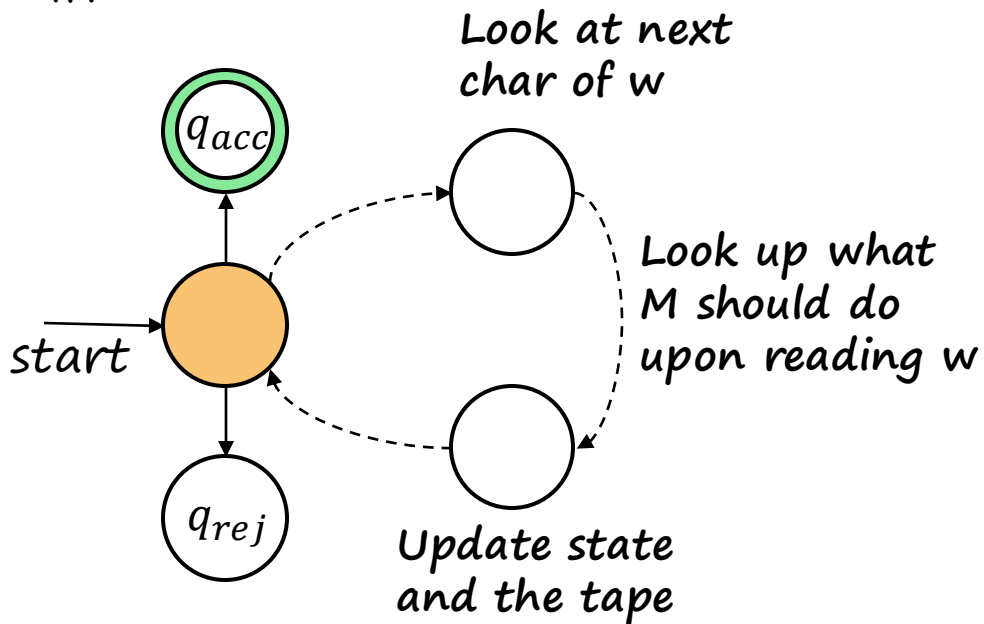
The Universal Turing Machine

Machine M



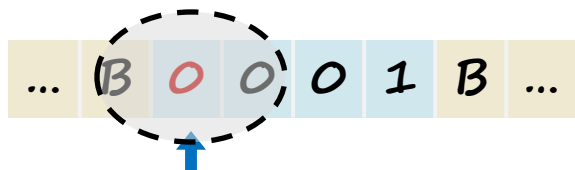
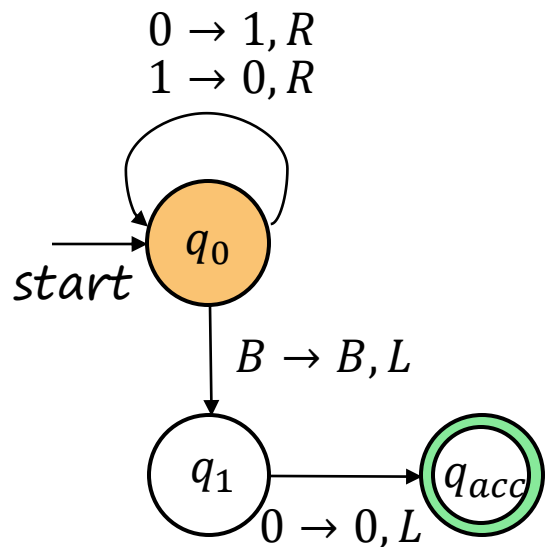
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U_{TM}

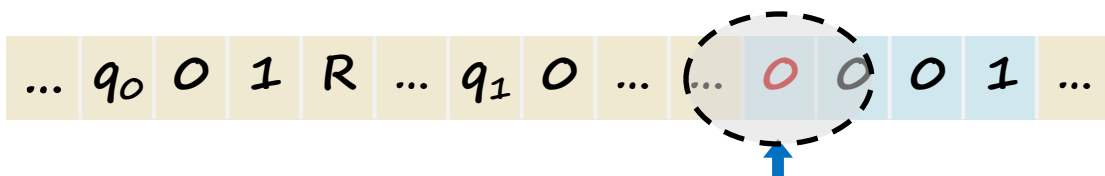
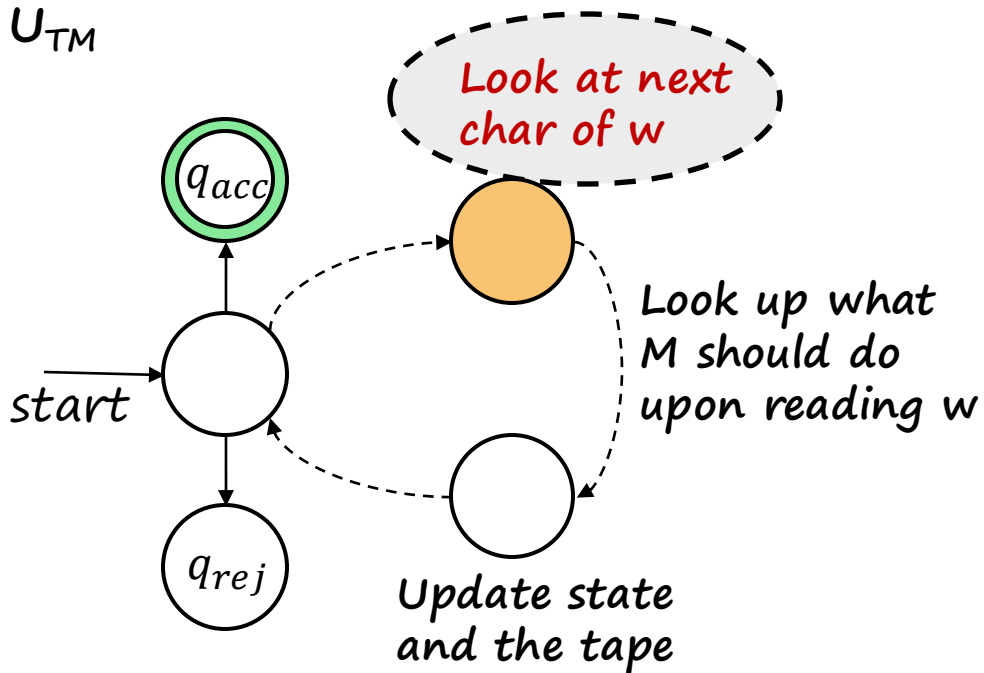


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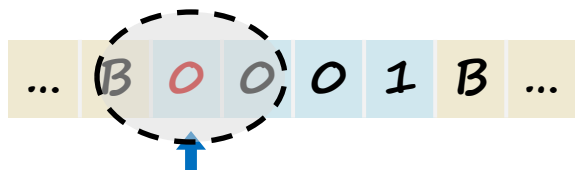
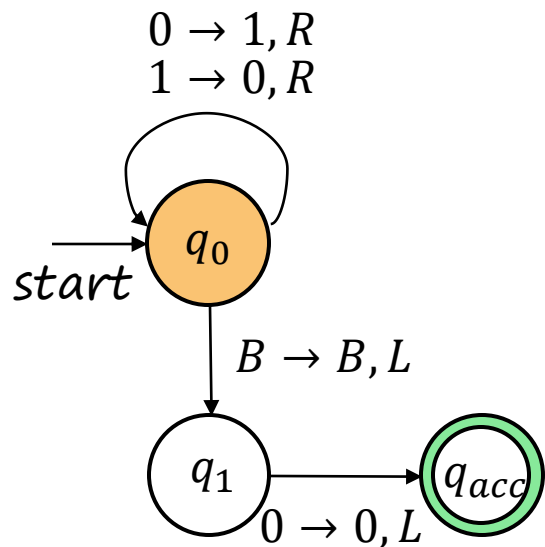


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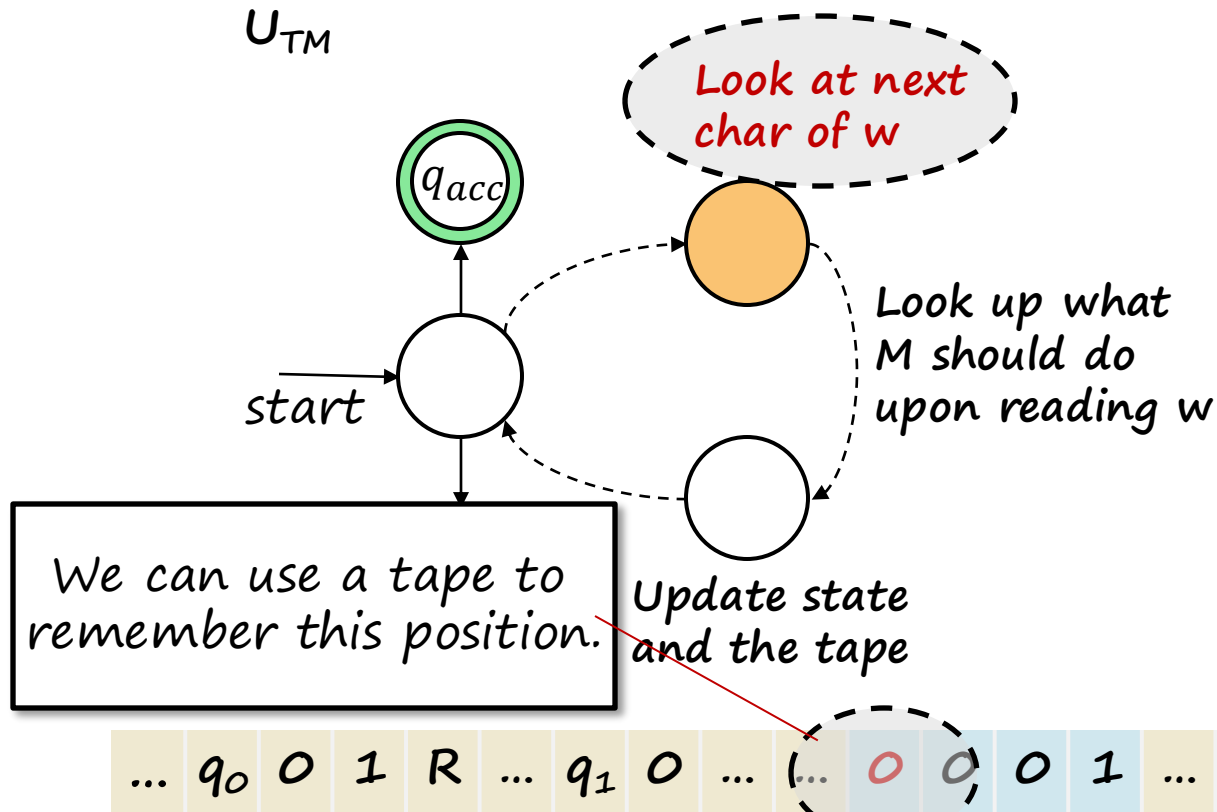


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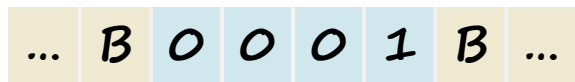
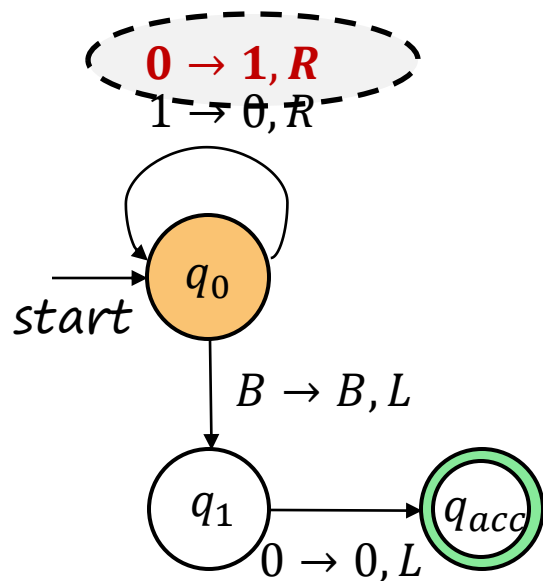


U_{TM}



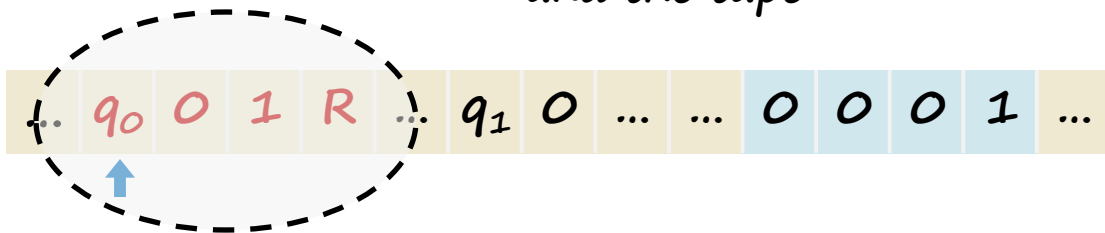
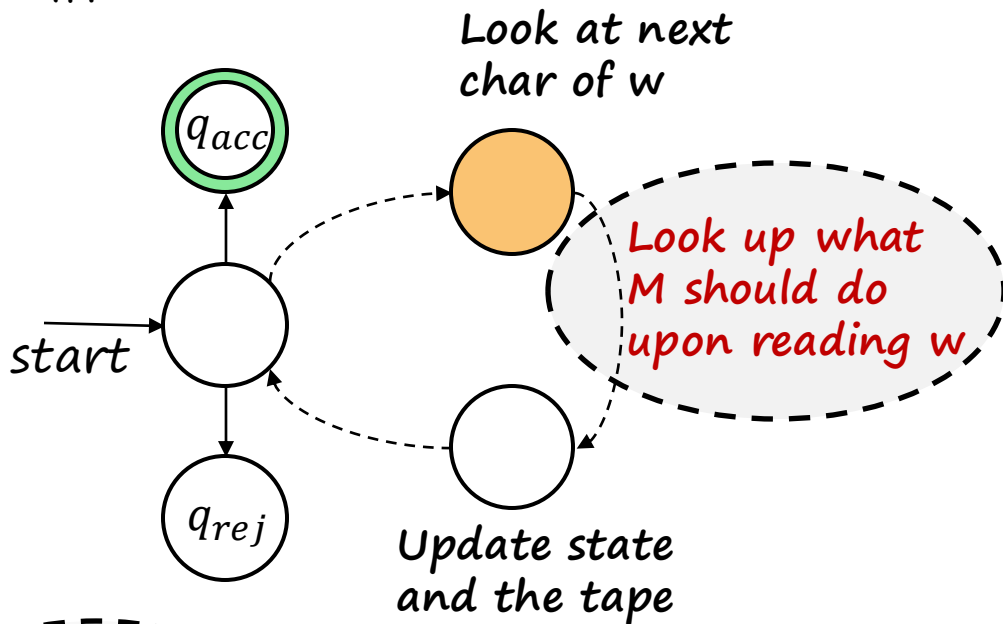
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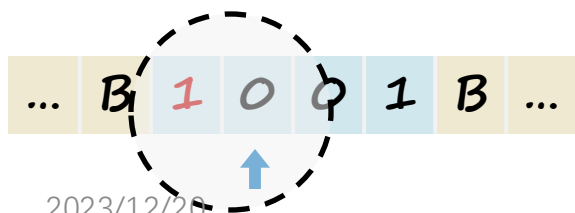
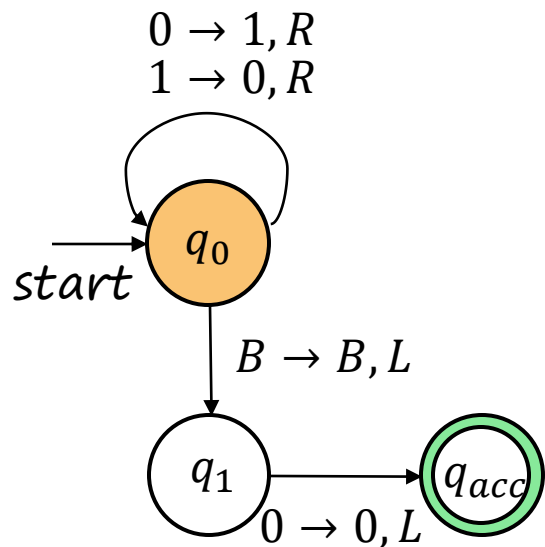
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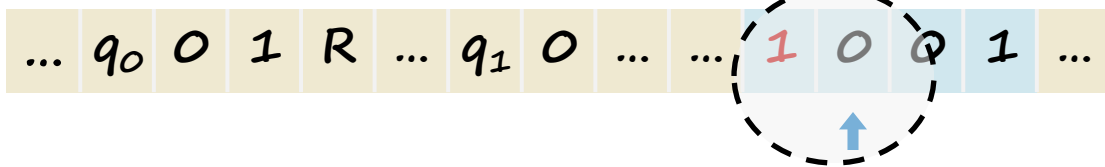
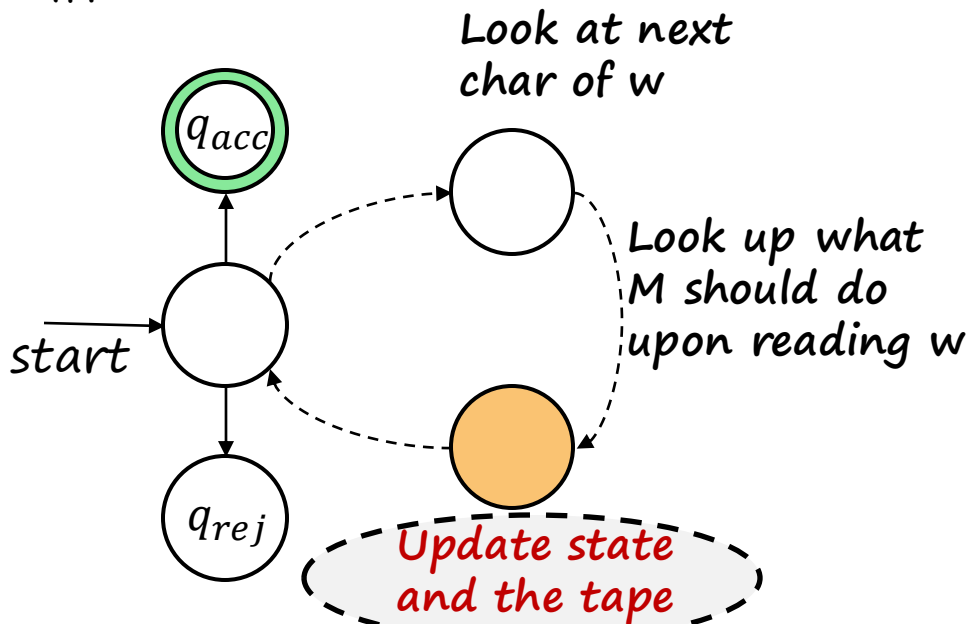


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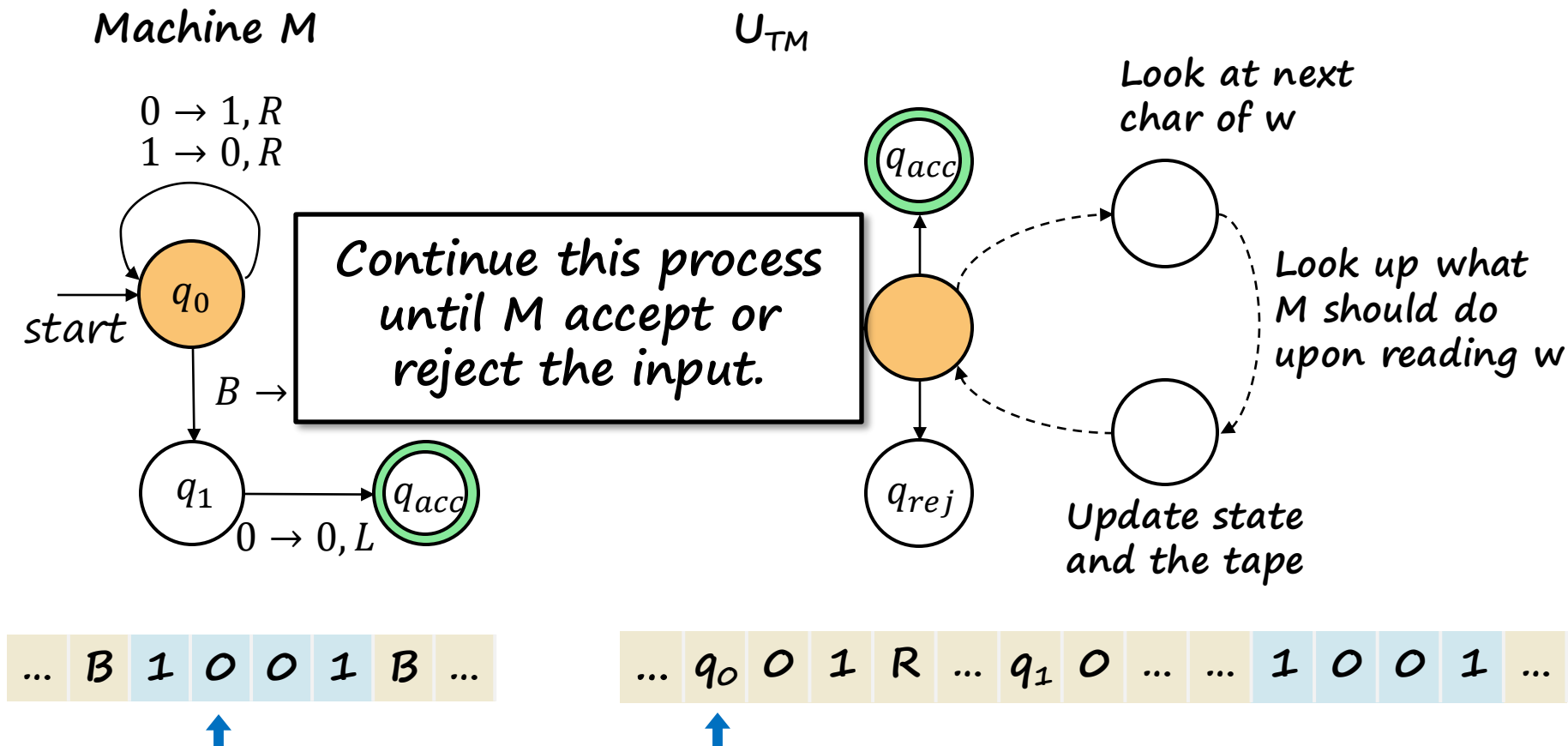
Machine M



U_{TM}



The Universal Turing Machine



The Language of U_{TM}

- Recall that the language of a TM is the set of all strings that TM accepts.
- U_{TM} when run on a string $\langle M, w \rangle$, where M is a TM and w is a string, will
 - Accept $\langle M, w \rangle$ if M accepts w
 - Reject $\langle M, w \rangle$ if M rejects w
 - Loop on $\langle M, w \rangle$ if M loops on w .

The Language of U_{TM}

- The **universal language**, denoted L_u , is the language of the U_{TM}

$$\begin{aligned} L_u &= L(U_{TM}) = \{\langle M, w \rangle \mid M \text{ is a TM and } M \text{ accepts } w\} \\ &= \{\langle M, w \rangle \mid M \text{ is a TM and } w \in L(M)\} \end{aligned}$$

- Useful fact:

$$\langle M, w \rangle \in L_u \Leftrightarrow M \text{ accepts } w$$

- Because $L_u = L(U_{TM})$, we know that $L_u \in RE$

The Language of U_{TM}

- If M accepts w , then we have:
 - U_{TM} accepts $\langle M, w \rangle$
 - U_{TM} accepts $\langle U_{TM}, \langle M, w \rangle \rangle$
 - U_{TM} accepts $\langle U_{TM}, \langle U_{TM}, \langle M, w \rangle \rangle \rangle$
 -

How we investigate computability?

2.1 Discussion on Problems

Part1. Intro & Set theory(I) : Basics & Formal Language.

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Part3. Capture Structures: Binary Relation & Function

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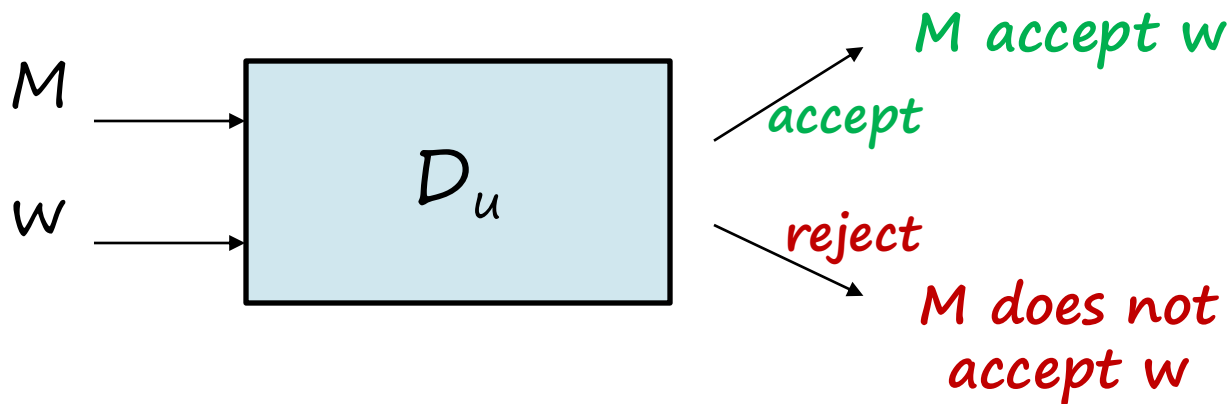
2.3 Discussion on computability

Part1. The Language of Turing Machine. R & RE

Part2. Undecidability.

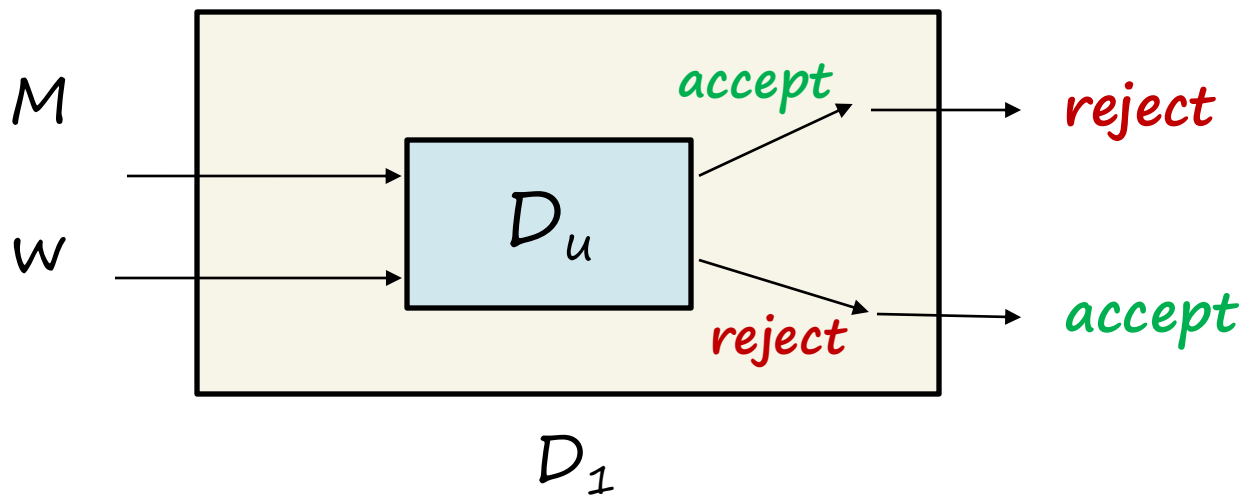
A Decider for L_u

- We know that $L_u \in \mathbf{RE}$, but whether $L_u \in \mathbf{R}$?
- That is, whether there is a decider D_u for L_u ?



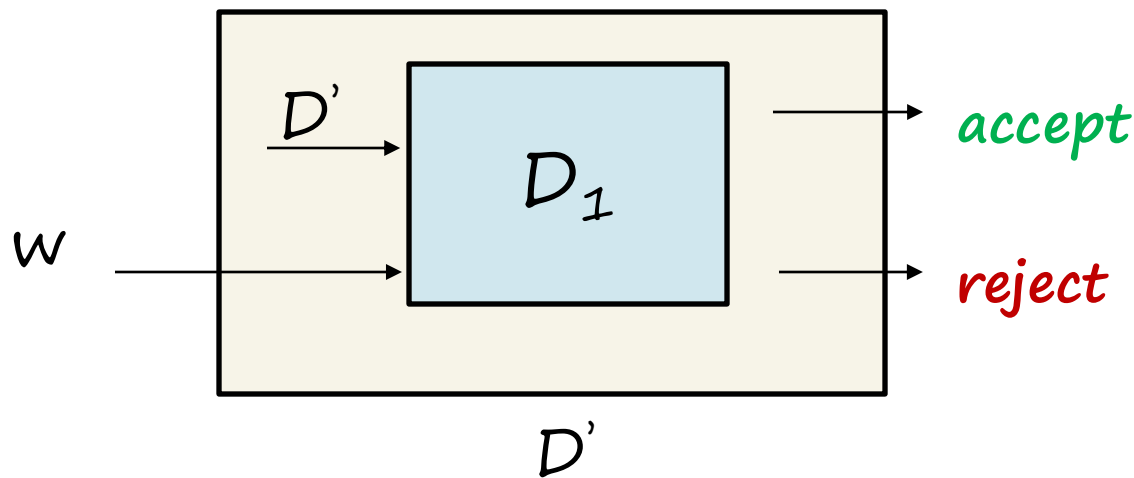
A Decider for L_u

- If D_u exists, we can build a new TM D_1 based on D_u
- D_1 takes a TM M and string w as inputs, and feeds them to D_u , then outputs the **reverse result** of D_u .



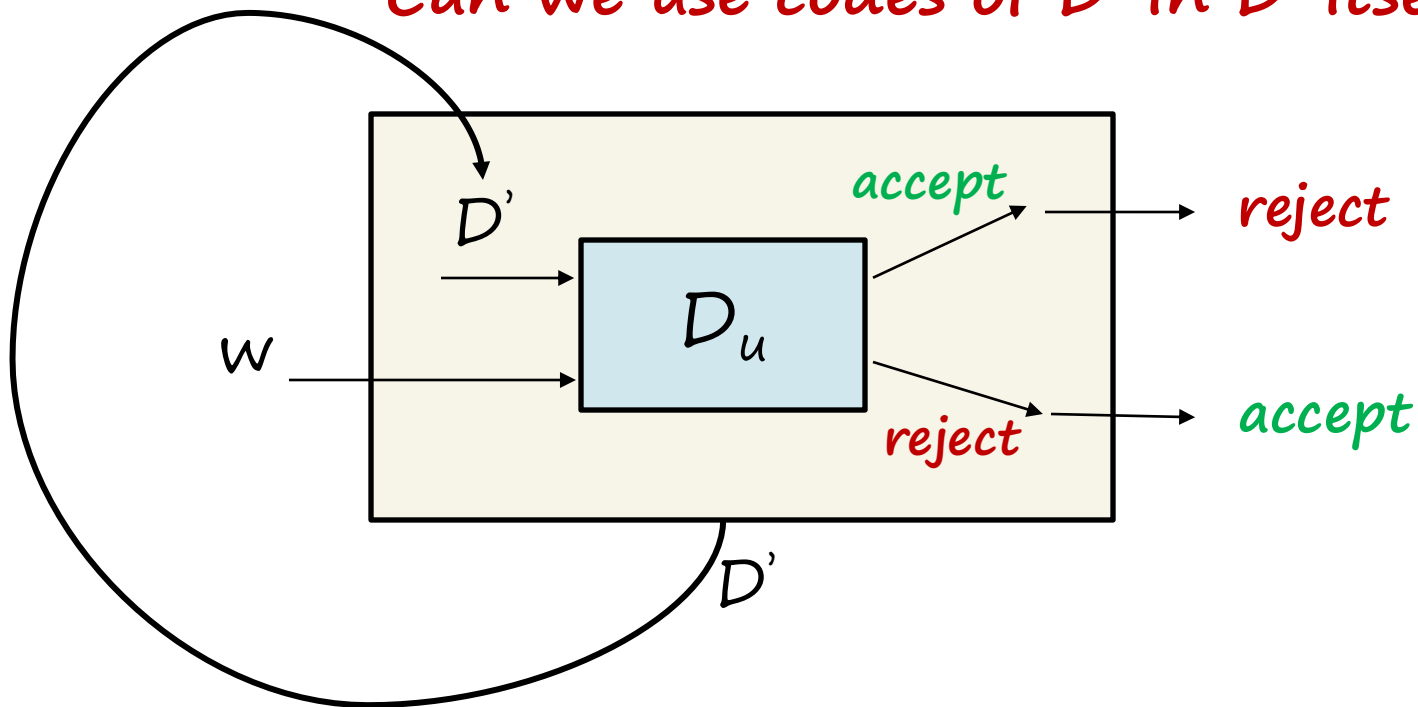
A Decider for L_u

- Then we can build another TM D' based on D_1
- D' only takes a string w as input, and feeds **its own code** and w to D_1 , then outputs the result of D_1 .

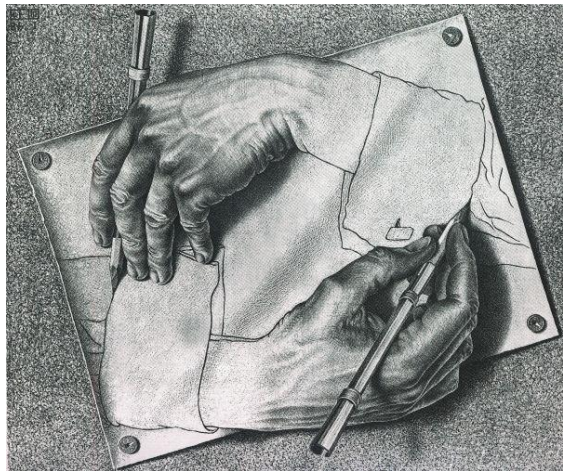


A Decider for L_u

Can we use codes of D' in D' itself?



Self-Reference



Sets that do not contain itself.

“This sentence is wrong.”

Self-Referential Software

- A **Quine** is a program that, when running, prints its own source code.
- Quines aren't allowed to just read the file containing their source code and print it out; that's cheating (and technically incorrect if someone changes that file!)
- How would you write such a program?

Example: Quine

```
#include <stdio.h>
char*f="#include <stdio.h>
char*f=%c%s%c;
main(){printf(f,34,f,34,10);}%"
main(){printf(f,34,f,34,10);}
```

<http://www.nyx.net/~gthompso/quine.htm>

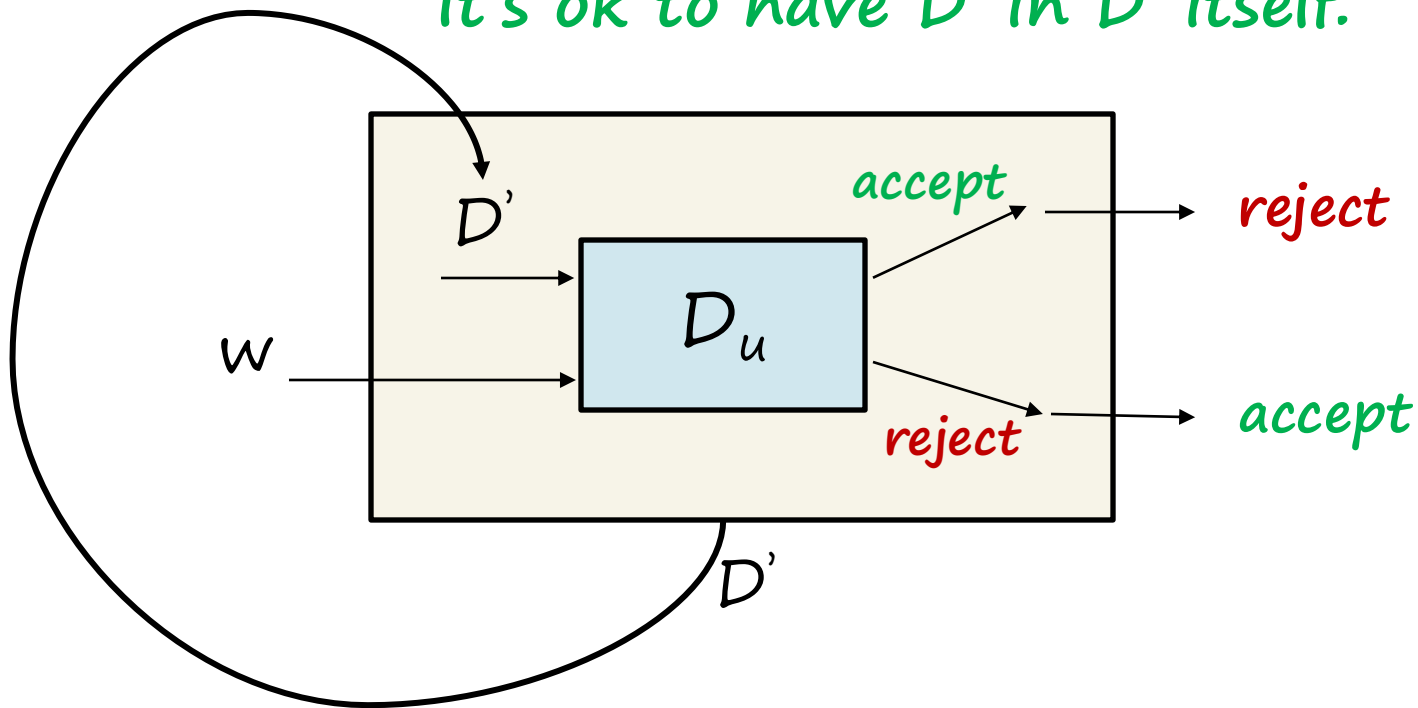
Self-Referential Software

- Going forward, assume that any program can be augmented to include a method called *mySource()* that returns a string representation of its source code.
- Then we can take the whole program codes as an input though we don't know what the whole program codes are, and even what we write now will change the final code itself.

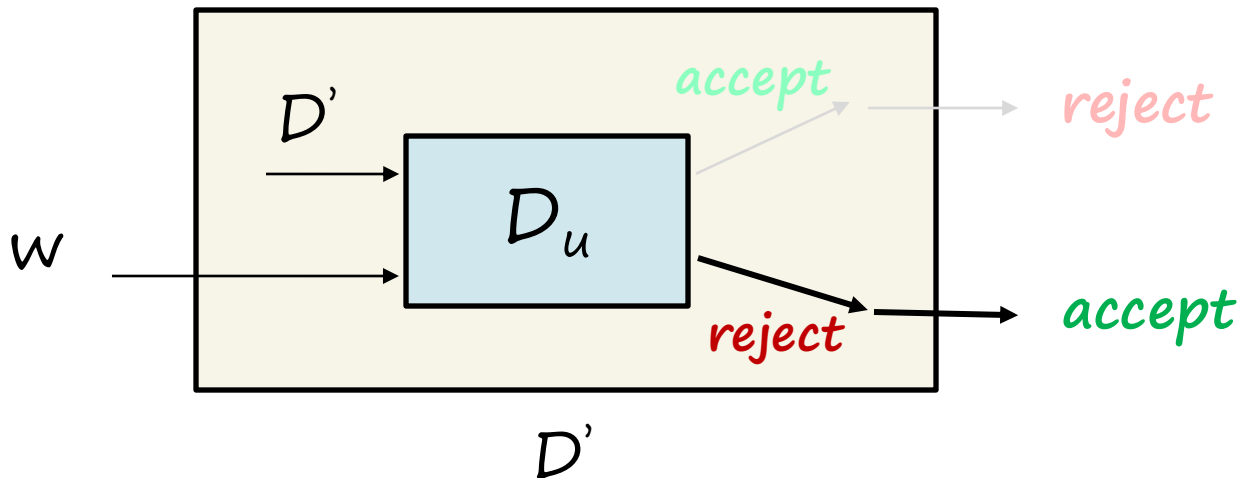
```
char * mySource(){...}  
int main(){  
    char * s = mySource();  
    .....  
}
```

A Decider for L_u

It's ok to have D' in D' itself.

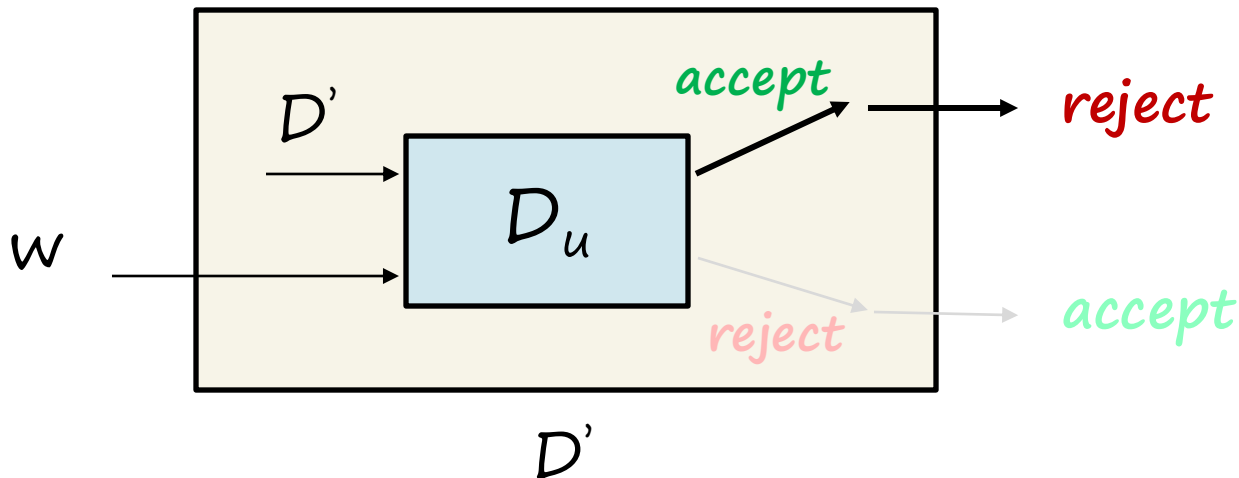


A Decider for L_u



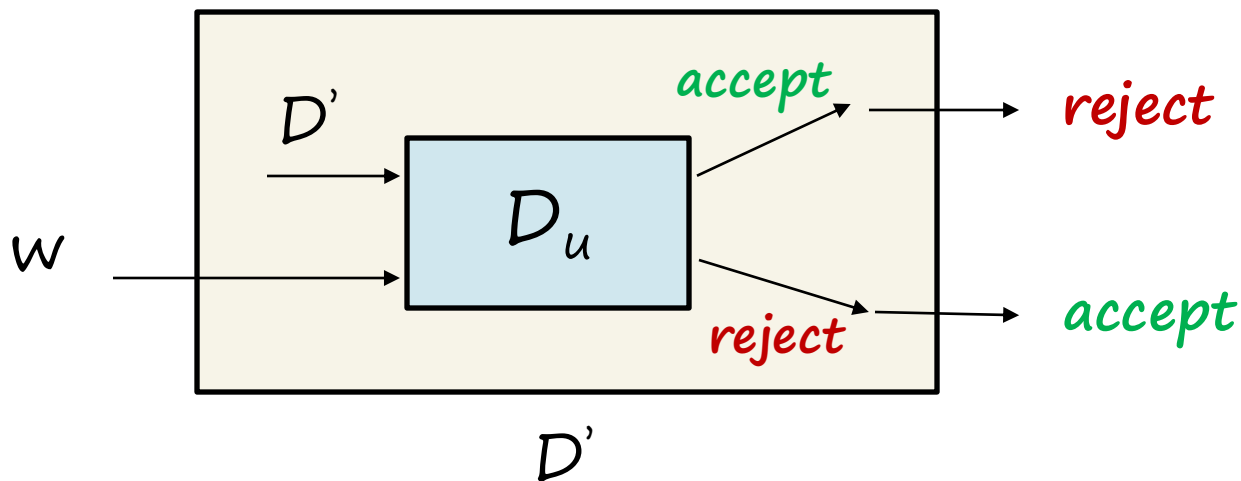
- If D' accepts w .
 - Then D_u rejects $\langle D', w \rangle$, which means that D' rejects w

A Decider for L_u



- If D' accepts w .
 - Then D_u rejects $\langle D', w \rangle$, which means that D' rejects w
- If D' rejects w .
 - Then D_u accepts $\langle D', w \rangle$, which means that D' accepts w

A Decider for L_u



D' accepts $w \Leftrightarrow D'$ rejects w

D_u can not exist!

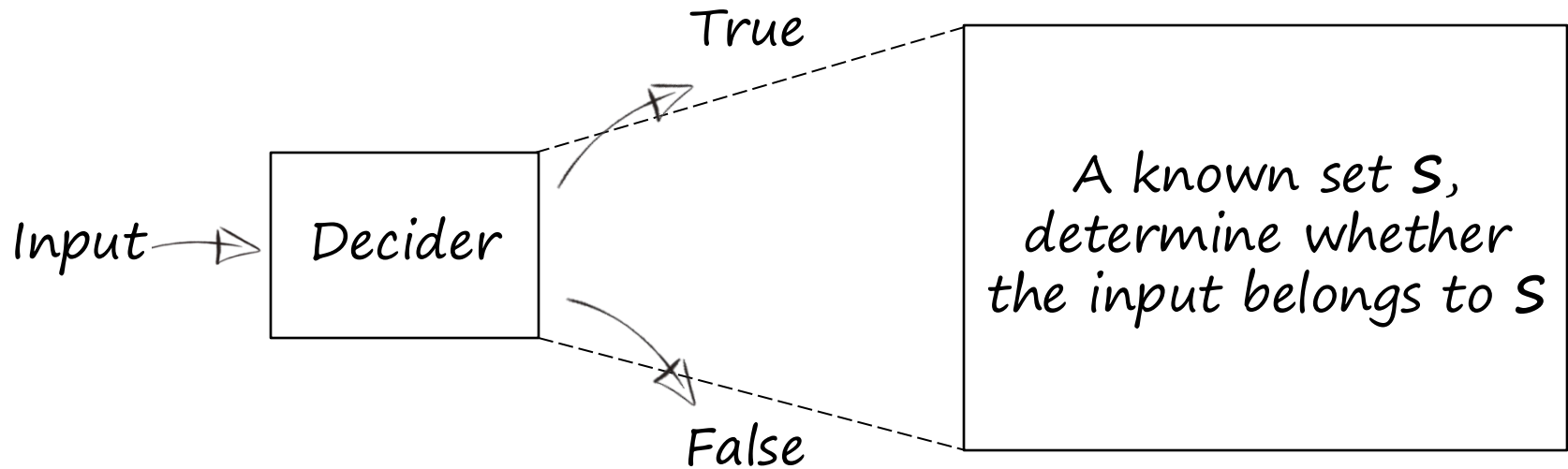

$$L_u \notin \mathbf{R}$$

- As there are no deciders for L_u , L_u is not in the class of $\mathbf{R}$.
- **Undecidable(不可判定):** there is **no algorithm** that can determine whether a program will definitely accept or reject an input.
- Intuition: The only general way to find out what a program will do is to run it.

Computable (可计算的)

- **A computational problem is computable if there is a procedure (or algorithm) for it which:**
 - Rigorously follows some finite instructions, requires no ingenuity.
 - Always finishes (terminates) after a finite number of steps.

Solve Decision Problem



Formal Language Theory

Alphabets are finite, non-empty sets of characters

Alphabet: Σ

$a, b, c \dots$
 $\wedge \vee \neg ()$

Characters are individual symbols

finite sequence

A string

$(a \vee b) \wedge (\neg b \vee c)$

Strings are finite sequence of characters

All strings

a, b, c
 $(a \vee b) \wedge (\neg b \vee c)$
 $\dots$

subset of Σ^*

A language

Languages are sets of strings.

$\vee \wedge abc$
 $a \wedge \neg a$
 $(a \vee b) \wedge (\neg b \vee c)$
 $\dots$

Set of all string : Σ^*

What problems can a computer solve?

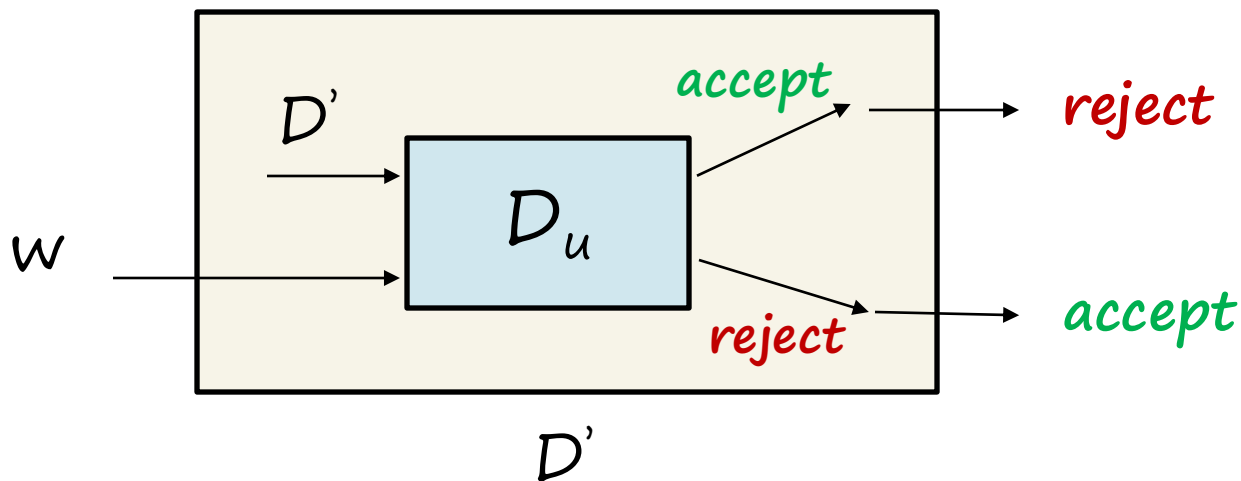


What languages can a computer recognize?



What languages can TM recognize?

An Undecidable Problem

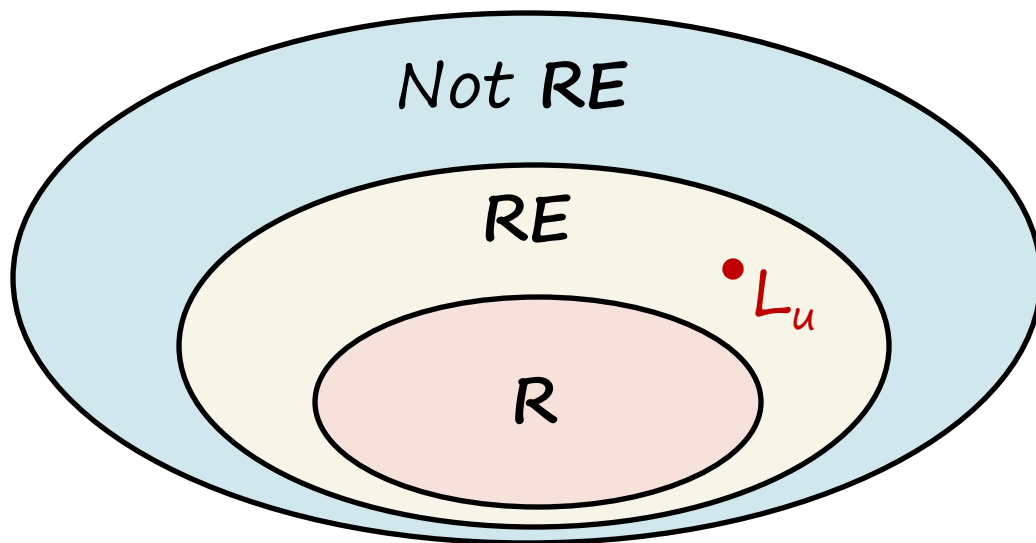


D' accepts $w \Leftrightarrow D'$ rejects w

D_u can not exist!

$R \neq RE$

- We found a language $L_u \in RE$, but $L_u \notin R$, which means $R \neq RE$
- Because $R \neq RE$, there is a difference between decidability and recognizability:



Turing Machine III & Undecidability

Zeyu Mi

2023-12-22



Adapted From:

IALC 9.1, 9.2

Stanford CS103 Turing Machine

Language that is Non-RE

Beyond RE

- The RE languages represent languages whose membership can be proven
 - If $w \in L$, we can prove that by a TM
 - So what does a non-RE language look like?
- This language cannot be recognized by any Turing machines

Numbering TMs

- Recall that a TM M can be encoded as a binary. In return, a binary can be interpreted as a TM.
 - Some binary may not be interpreted as a TM (illegal format), we interpret that binary as a TM without any transition function and accepting no string
 - TM code begins with 0's, and we can add a prefix 1 to the code of TM as the number of that TM
 - So the numerical values of different TMs are different.

Numbering TMs

Turning Machine



encode
⇌
decode

Code for TM

001010010100

add
prefix
⇌
remove
prefix

Number for TM

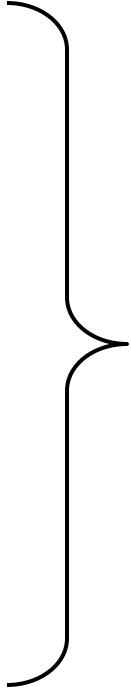
1001010010100
=4756

$M=4756$

Numbering TMs

- We assign each TM a positive integer, and each positive integer has one TM corresponding to it.
 - We say that the i th TM M_i is the TM whose number is i .
- So we've build a bijection between the set of all the TMs and the set of positive integer. Then we can list all the TMs one by one.

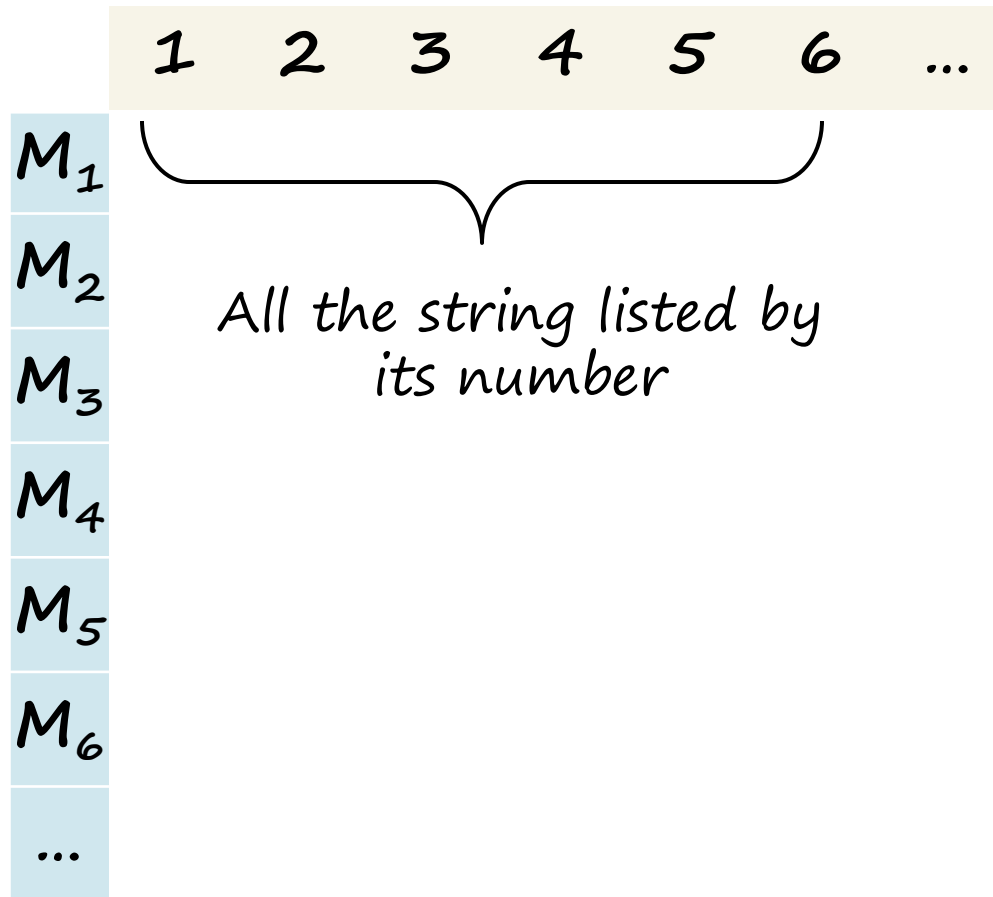
M_1
 M_2
 M_3
 M_4
 M_5
 M_6
...



*All Turing Machines
listed by its number*

Numbering All the String

- Each binary string represents a number
- We add a prefix 1 to each binary string to prevent the following condition
 - “01” and “001” are two different binary strings, but they represent the same number (1)
- w_i denotes the string with number i .
 - $w_1 = \epsilon, w_2 = 0, w_3 = 1, w_4 = 00$



	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...

1 in row i column j
means that M_i
accepts w_j .

0 means "not
accept".

M_3 accepts $w_4(00)$

M_6 does not
accept $w_6(10)$

	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...

Row i represents the language of M_i

$$L(M_4) = \{w_1, w_2, w_4, w_6\}$$

	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...



	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...

Flip this language.
Swap what's included
and what's excluded



	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...

No TM has this behavior!



Recall: Cardinality of Power Set

- Does $|\mathcal{P}(S)| > |S|$ for all infinite sets S ?
- For an infinite set $S = \{x_0, x_1, x_2, \dots\}$, we assume that $\mathcal{P}(S) \approx S$
- It means we can pair up the elements of S and the **subset of S** without leaving anything out.

$$x_0 \longleftrightarrow \{x_0, x_2, x_4, \dots\}$$

$$x_1 \longleftrightarrow \{x_3, x_5, \dots\}$$

$$x_2 \longleftrightarrow \{x_0, x_1, x_2, x_5, \dots\}$$

$$x_3 \longleftrightarrow \{x_1, x_4, \dots\}$$

$$x_4 \longleftrightarrow \{x_2, \dots\}$$

$$x_5 \longleftrightarrow \{x_0, x_4, x_5, \dots\}$$

$$\dots \longleftrightarrow \{\dots\}$$

x_0	x_1	x_2	x_3	x_4	x_5	$\dots$
-------	-------	-------	-------	-------	-------	---------

$$x_0 \longleftrightarrow \{x_0, x_2, x_4, \dots\}$$

$$x_1 \longleftrightarrow \{x_3, x_5, \dots\}$$

$$x_2 \longleftrightarrow \{x_0, x_1, x_2, x_5, \dots\}$$

$$x_3 \longleftrightarrow \{x_1, x_4, \dots\}$$

$$x_4 \longleftrightarrow \{x_2, \dots\}$$

$$x_5 \longleftrightarrow \{x_0, x_4, x_5, \dots\}$$

$$\dots \longleftrightarrow \{\dots\}$$

		x_0	x_1	x_2	x_3	x_4	x_5	...
x_0	$\longleftrightarrow \{$	$x_0,$		$x_2,$		$x_4,$		...
x_1	$\longleftrightarrow \{$				$x_3,$		$x_5,$	...
x_2	$\longleftrightarrow \{$	$x_0,$		$x_2,$			$x_5,$	...
x_3	$\longleftrightarrow \{$		$x_1,$			$x_4,$		...
x_4	$\longleftrightarrow \{$			$x_2,$				...
x_5	$\longleftrightarrow \{$	$x_0,$				$x_4,$	$x_5,$	...
...	$\longleftrightarrow \{$	...	...	...	...	...	...	...

	x_0	x_1	x_2	x_3	x_4	x_5	...
$x_0 \longleftrightarrow \{$	$x_0,$		$x_2,$		$x_4,$		$\dots \}$
$x_1 \longleftrightarrow \{$				$x_3,$		$x_5,$	$\dots \}$
$x_2 \longleftrightarrow \{$	$x_0,$		$x_2,$			$x_5,$	$\dots \}$
$x_3 \longleftrightarrow \{$		$x_1,$			$x_4,$		$\dots \}$
$x_4 \longleftrightarrow \{$			$x_2,$				$\dots \}$
$x_5 \longleftrightarrow \{$	$x_0,$				$x_4,$	$x_5,$	$\dots \}$
$\dots \longleftrightarrow \{$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots \}$
$\{$ $x_0,$ $x_2,$ $x_5,$ $\}$							

		x_0	x_1	x_2	x_3	x_4	x_5	...			
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}	
x_1	$\longleftrightarrow$	{			$x_3,$		$x_5,$		...	}	
x_2	$\longleftrightarrow$	{	$x_0,$		$x_2,$			$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$			$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$					...	}
x_5	$\longleftrightarrow$	{	$x_0,$				$x_4,$	$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	...	}

Flip this set.
Swap what's
included and
what's excluded

{		$x_1,$		$x_3,$	$x_4,$		...	}
---	--	--------	--	--------	--------	--	-----	---

		x_0	x_1	x_2	x_3	x_4	x_5	...				
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}		
x_1	$\longleftrightarrow$	{				$x_3,$		$x_5,$		...	}	
x_2	$\longleftrightarrow$	{	$x_0,$			$x_2,$			$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$				$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$						...	}
x_5	$\longleftrightarrow$	{	$x_0,$				$x_4,$		$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	...	...	}

Which element
is paired with
this set?

{		$x_1,$		$x_3,$	$x_4,$		...	}
---	--	--------	--	--------	--------	--	-----	---

		x_0	x_1	x_2	x_3	x_4	x_5	...
x_0	$\longleftrightarrow$	$\{$	$x_0,$		$x_2,$		$x_4,$	$\dots \}$
x_1	$\longleftrightarrow$	$\{$			$x_3,$		$x_5,$	$\dots \}$
x_2	$\longleftrightarrow$	$\{$	$x_0,$		$x_2,$		$x_5,$	$\dots \}$
x_3	$\longleftrightarrow$	$\{$		$x_1,$		$x_4,$		$\dots \}$
x_4	$\longleftrightarrow$	$\{$			$x_2,$			$\dots \}$
x_5	$\longleftrightarrow$	$\{$	$x_0,$			$x_4,$	$x_5,$	$\dots \}$
...	$\longleftrightarrow$	$\{$	...	...	...	...	...	$\dots \}$
$\{$ $x_1,$ $x_3,$ $x_4,$ $\dots \}$								

		x_0	x_1	x_2	x_3	x_4	x_5	...		
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}
x_1	$\longleftrightarrow$	{			$x_3,$		$x_5,$		...	}
x_2	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$		$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$				...	}
x_5	$\longleftrightarrow$	{	$x_0,$			$x_4,$	$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	}
{ $x_1,$ $x_3,$ $x_4,$... }										

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		x_0	x_1	x_2	x_3	x_4	x_5	...			
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}	
x_1	$\longleftrightarrow$	{				$x_3,$		$x_5,$		...	}
x_2	$\longleftrightarrow$	{	$x_0,$		$x_2,$			$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$			$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$					...	}
x_5	$\longleftrightarrow$	{	$x_0,$				$x_4,$	$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	...	}
{ $x_1,$ $x_3,$ $x_4,$... }											

		x_0	x_1	x_2	x_3	x_4	x_5	...	
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$	...	}
x_1	$\longleftrightarrow$	{			$x_3,$		$x_5,$	...	}
x_2	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_5,$	...	}
x_3	$\longleftrightarrow$	{		$x_1,$		$x_4,$		...	}
x_4	$\longleftrightarrow$	{			$x_2,$			...	}
x_5	$\longleftrightarrow$	{	$x_0,$			$x_4,$	$x_5,$	...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	}
{ $x_1,$ $x_3,$ $x_4,$...									

2/22

		x_0	x_1	x_2	x_3	x_4	x_5	...				
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}		
x_1	$\longleftrightarrow$	{				$x_3,$		$x_5,$		...	}	
x_2	$\longleftrightarrow$	{	$x_0,$			$x_2,$			$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$			$x_4,$				...	}
x_4	$\longleftrightarrow$	{			$x_2,$						...	}
x_5	$\longleftrightarrow$	{	$x_0,$				$x_4,$	$x_5,$			...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	...	...	}
{ $x_1,$ $x_3,$ $x_4,$... }												

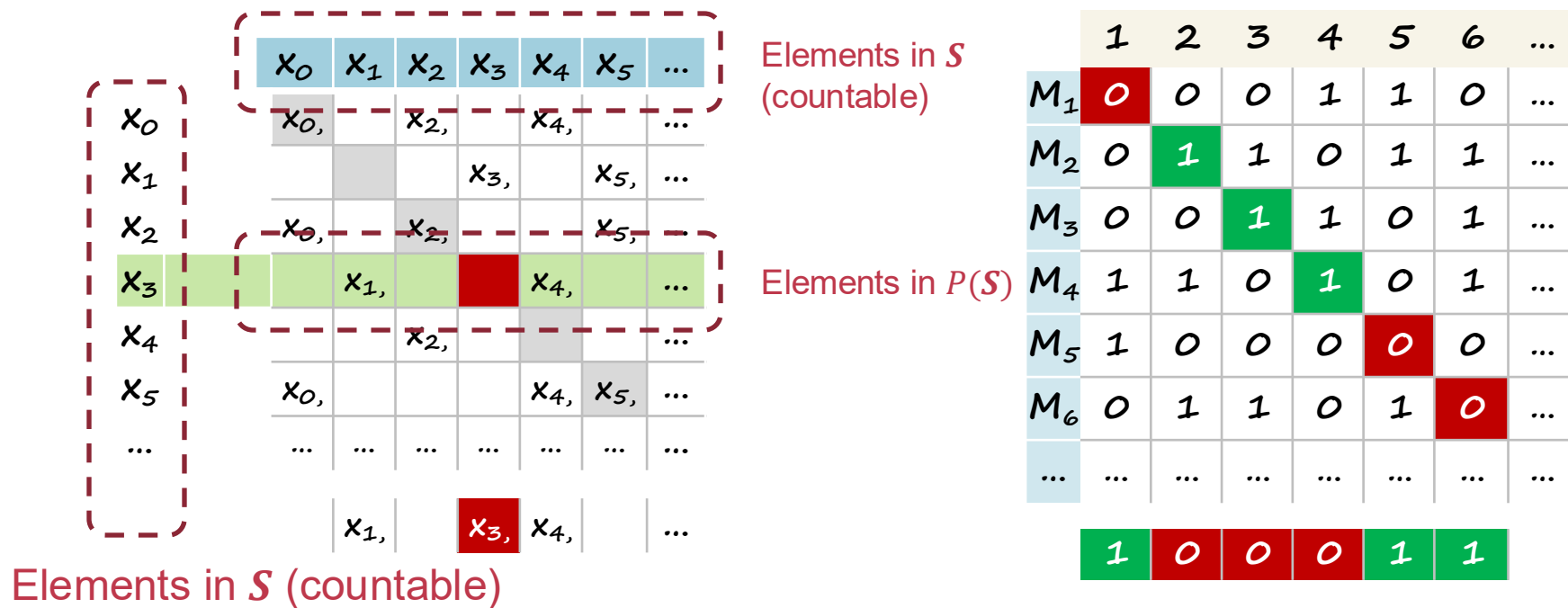
		x_0	x_1	x_2	x_3	x_4	x_5	...		
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}
x_1	$\longleftrightarrow$	{			$x_3,$		$x_5,$		...	}
x_2	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$		$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$				...	}
x_5	$\longleftrightarrow$	{	$x_0,$			$x_4,$	$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	}
{ $x_1,$ $x_3,$ $x_4,$... }										

		x_0	x_1	x_2	x_3	x_4	x_5	...				
x_0	$\longleftrightarrow$	{	$x_0,$		$x_2,$		$x_4,$		...	}		
x_1	$\longleftrightarrow$	{				$x_3,$		$x_5,$		...	}	
x_2	$\longleftrightarrow$	{	$x_0,$			$x_2,$			$x_5,$		...	}
x_3	$\longleftrightarrow$	{		$x_1,$				$x_4,$			...	}
x_4	$\longleftrightarrow$	{			$x_2,$						...	}
x_5	$\longleftrightarrow$	{	$x_0,$				$x_4,$		$x_5,$		...	}
...	$\longleftrightarrow$	{	...	...	...	...	...	...	...	...	...	}

No element can be paired with this set!!!

{		$x_1,$		$x_3,$	$x_4,$		...	}
---	--	--------	--	--------	--------	--	-----	---

Comparing two diagonal methods



Conclusion: No match between S and $P(S)$

Comparing two diagonal methods

	x_0	x_1	x_2	x_3	x_4	x_5	...
x_0	$x_0,$		$x_2,$		$x_4,$		...
x_1				$x_3,$		$x_5,$	...
x_2	$x_0,$		$x_2,$			$x_5,$	...
x_3		$x_1,$			$x_4,$		...
x_4			$x_2,$				...
x_5	$x_0,$				$x_4,$	$x_5,$	...
...	...	...	...	...	...	...	...

$x_1,$ $x_3,$ $x_4,$...

All strings
(countable)

	1	2	3	4	5	6	...
M_1	0	0	0	1	1	0	...
M_2	0	1	1	0	1	1	...
M_3	0	0	1	1	0	1	...
M_4	1	1	0	1	0	1	...
M_5	1	0	0	0	0	0	...
M_6	0	1	1	0	1	0	...
...	...	...	...	...	...	...	...

1 0 0 0 1 1

All TMs
(countable)

A behavior of an automata
(corresponding to a language)

Conclusion: Turing machines can not recognize all languages

Diagonalization Language

- The language L_d , the diagonalization language, is the set of string w_i such that w_i is not in $L(M_i)$

$$L_d = \{w_i | w_i \notin L(M_i)\}$$

- Notice that the code for M_i is just w_i , so we can write L_d in a more interesting way:

$$L_d = \{M | M \notin L(M)\}$$

- The diagonalization language is all the codes of Turing machine which does not accept its own code.

Where We're Going

- A string is a sequence of characters.
 - There are at most as many programs as there are strings.
 - There are at least as many problems as there are sets of strings.
- **From Cantor's Theorem, we know that there are more sets of strings than strings**

$$|\text{Programs}| \leq |\text{Strings}| < |\text{Set of strings}| \leq |\text{Problems}|$$

Where We Are

- Now can we answer the question:

What problems can a computer solve?

- Unfortunately we can only tell there exist some unsolvable(undecidable) problems. But we cannot know exactly what problems can be solved.
 - Actually there's little understanding about the sufficient condition for a problem to be computable.

Where We Are

- Knowing some problem is undecidable means:
 - Hold back our ambition of finishing an impossible task. (The universal program verifier does not exist.)
 - Then try to find another decidable problem to solve. (But we can build a verifier for specific programs.)
 - And it is interesting itself.

How we investigate computability?

2.1 Discussion on Problems

Part1. Intro & Set theory(I) : Basics & Formal Language.

Part2. Set Theory(II): Axiom system & Cardinality.

Part3. Capture Structures: Binary Relation & Function

2.2 Discussion on Computation

Part1. Turing Machine Basics.

Part2. Variants of Turing Machine. Church-Turing Thesis.

2.3 Discussion on computability

Part1. The Language of Turing Machine. R & RE

Part2. Undecidability.